ENLIVE: An Efficient Nonlinear Method for Calibrationless and Robust Parallel Imaging

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Abstract: We propose an extension to Regularized Nonlinear Inversion (NLINV) which simultaneously reconstructs multiple images and sets of coil sensitivity profiles. It can be shown that this method is related to a lifted formulation of blind deconvolution with nuclear-norm regularization, which inherently promotes low-rank solutions. The method, termed ENLIVE (Extended Non-Linear InVersion inspired by ESPIRiT) is more robust against data inconsistencies and model violations. This is demonstrated in initial experiments.

Zusammenfassung: Wir stellen eine neue Methode basierend auf einer Erweiterung von Regularized Nonlinear Inversion (NLINV) vor. Diese Methode rekonstruiert gleichzeitig mehrere Bilder und Sätze von Spulenprofilen. Diese Methode hat eine enge Verbindung zu einer Formulierung von blinder Entfaltung mit Regularisierung der Nuklearnorm. Diese Formulierung bevorzugt inhärent Lösung mit niedrigem Rang. Die Methode, genannt ENLIVE (Extended Non-Linear InVersion inspired by ESPIRiT), ist robust gegen Inkonsistenz in den Daten und gegen Modellverletzungen. Dies demonstrieren wir in ersten Experimenten.

Motivation

ESPIRiT (1) has been shown to provide robustness against data inconsistencies similar to GRAPPA (2). This robustness is possible through a relaxed model which uses multiple images and coil sensitivity profiles. ESPIRiT, however, is not applicable to calibrationless data and needs preprocessing steps to work with non-Cartesian data. NLINV (3), on the other hand, can be applied directly in these settings. The approach presented here aims to combine these advantages. Additionally, the presented formulation can be related to a convex problem which produces solutions with low rank.

Theory

Given measurements y_i from multiple coils, ENLIVE recovers k images m^i and k sets coil profiles c_j^i by solving the following regularized nonlinear optimization problem (Eq. 1):

$$\operatorname*{arg\,min}_{m^i, c_j^i} \sum_j \|y_j - \mathcal{P}\mathcal{F}\{\sum_i^k c_j^i m^i\}\|_2^2 + \frac{\alpha}{2}\sum_{i,j}(\|W c_j^i\|_2^2 + \|m^i\|_2^2)$$

where α is a regularization parameter, F is a two or three dimensional Fourier transform, P is the projection onto the measured trajectory (or the acquired pattern in Cartesian imaging) and W is a weighting matrix penalizing high frequencies in the coil profiles. When allowing only one map (i.e. k=1), ENLIVE coincides with the NLINV problem. We solve this problem using the iteratively regularized Gauss-Newton method.

Since the problem is exactly symmetric in the maps, we break this symmetry by applying Gram-Schmidt orthogonalization to the coil profiles after each Newton step. For this, the coil profiles of each set are treated as stacked one-dimensional vectors.

In the following we explain how the quadratically regularized bilinear problem of ENLIVE corresponds to a convex problem using nuclear-norm regularization. The convex formulation is not used for computation, but instead explains why ENLIVE produces solutions with low rank.

NLINV (k=1) can be understood as blind multi-channel deconvolution in k-space. This problem can be lifted into a linear inverse problem in terms of the outer product of uv^T, where u corresponds to m and v is a stacked vector of the weighted coil sensitivity profiles $Wc_j=\hat{c}_j$. The matrix $X=uv^T$ is obviously rank 1. Relaxing this rank constraint corresponds to choosing $X=UV^T$, where $U \in \mathcal{C}^{N\times k}$ is formed by the k vectorized images m^i and $V \in \mathcal{C}^{M\times k}$ is formed by the stacked weighted coil sensitivity profiles $\hat{c}_j^i$. Together with a linear opertator A_j with which maps UV^T onto $\mathcal{P}\mathcal{F}\{\sum_i^k c_j^i m^i\}$, this can be written as a fully convex formulation (Eq. 2):

$$\operatorname*{arg\,min}_X \sum_j \|y_j - \mathcal{A}_j\{X\}\|_2^2 + \alpha\|X\|_\star$$

It has been shown (4, 5) that, if Eq. 2 has a solution of rank ≤k, then that solution corresponds to a solution of the original deconvolution problem (Eq. 1). Since the regularization with the nuclear norm in Eq. 2 promotes low-rank solutions, this explains why ENLIVE, as formulated in Eq. 1, also produces solutions with low rank.

Methods:

We applied ENLIVE to the same dataset used in (1), which is a retrospectively two-fold undersampled 2D spin-echo dataset, which TR/TE=550/14 ms, FA=90°, matrix size: 320x168, slice thickness 3 mm and an FOV of 200x150 mm^2 acquired at 1.5 T using an 8-channel head coil. In this acquisition, the FOV is smaller than the subject's head in the lateral direction, which leads to infolding artifacts in traditional SENSE reconstruction. This dataset was reconstructed using ENLIVE allowing 1 to 4 sets of maps. The reconstructions using 1 and 2 maps were also compared to ESPIRiT.

Results

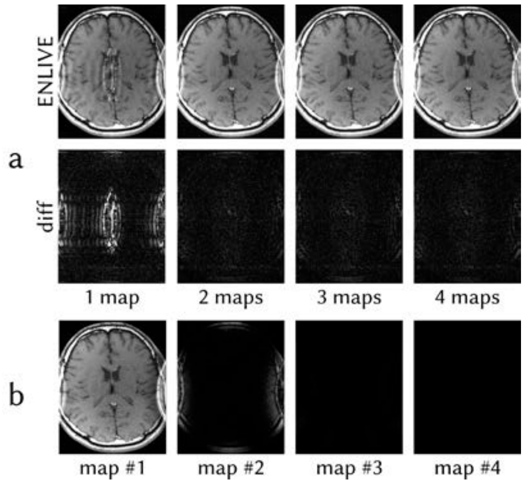

Fig. 1: (a) ENLIVE reconstructions using 1, 2, 3, and 4 sets of maps. The bottom row shows difference images to a fully-sampled reference. Using only a single map, a central infolding artifact is visible in both the reconstruction and the difference image. Using more than 1 map resolves this artifact. There is, however, no discernable difference between using 2 to 4 maps. (b): Individual map images of the reconstruction using 4 maps. The first map contains most of the image, the second map contains the infolded regions. Maps 3 and 4 are essentially zero.

As shown in Fig. 1a, reconstruction using a single map (NLINV) leads to infolding artifacts. Using multiple maps, this artifact can be avoided. There is, however, no benefit from using more than 2 maps. This is explained through Fig. 1b, which shows the individual map images of the reconstruction with 4 maps: Only the first two maps contain signal, maps 3 and 4 are essentially zero. Since no thresholding is used, they cannot be exactly zero. Fig. 2 shows a comparison between ENLIVE and ESPIRiT reconstructions of the same dataset using 1 and 2 maps. Using 1 map, both methods show artifacts which can be resolved by using 2 maps. Furthermore the image quality of both methods is similar.

Discussion

An FOV smaller than the imaged object causes infolding artifacts in traditional SENSE reconstructions. These artifacts appear because the infolded regions on the side cannot be explained using a single set of coil profiles, thereby violating the model. Using a relaxed model with multiple images and coil profiles allows artifact free reconstruction. This was tested with both ENLIVE and ESPIRiT, with both showing comparable image quality. Furthermore, using more than the required number of maps (here: 2) does not impair image quality, since ENLIVE will leave this additional maps empty. This shows that ENLIVE automatically produces solutions with low rank.

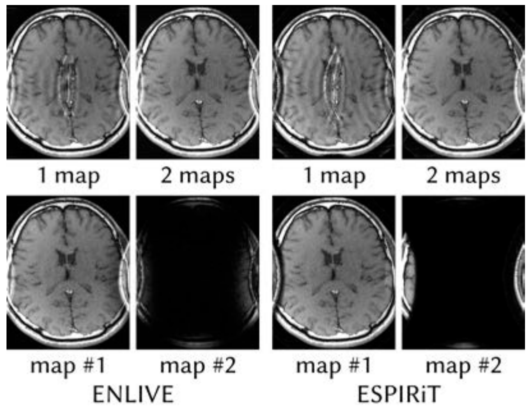

Fig. 2: Comparison of ENLIVE reconstruction using 1 and 2 maps to ESPIRiT using 1 and 2 maps. The single map reconstructions of both methods show a central infolding artifact, which can be avoided using 2 maps. Using 2 maps, image quality of both methods is similar. The bottom row shows the individual map images. For both methods, map 1 contains most of the image while map 2 contains the infolded region.

Conclusion

Here, we propose ENLIVE, a nonlinear method for parallel imaging inspired by ESPIRiT. By relating it to a convex formulation with nuclear-norm regularization, it can be explained why ENLIVE produces low-rank solutions. This was also shown in initial experiments. Furthermore, ENLIVE demonstrates ESPIRiT-like robustness in imaging.

References

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